

---

***NORTHROP GRUMMAN***

The logo features the company name in a bold, italicized, blue sans-serif font. A thin blue horizontal line is positioned directly beneath the text. Below this line, a curved blue swoosh starts under the 'N' and sweeps upwards and to the right, ending under the 'M'.

# **A Resolution Enhancement Technique for Laser Gyros**

**J.G. Mark and D.A. Tazartes**



### **D.A. Tazartes**

Mr. Daniel Tazartes is Director, Advanced Technology at Northrop Grumman's Navigation Systems Division. Mr. Tazartes has been at Northrop Grumman and the former Litton Guidance and Control Systems for the past 22 years. During that time he has successfully introduced several generations of new instrument technologies into inertial and integrated navigation systems. Mr. Tazartes holds over 50 issued U.S. patents in the fields of laser, fiber optic, and Microelectromechanical Sensors (MEMS), as well as in control algorithms, electronics, and signal processing for inertial sensors and systems. He has published numerous articles and reference texts on optical sensors for navigation and on strapdown navigation technology.

At Litton Guidance and Control Systems, Mr. Tazartes helped develop the ring-laser-gyro-based strapdown navigation systems. He optimized both the instrument and system designs to make best use of optical gyroscopes in inertial and inertial-GPS navigation systems. As laser-gyro-based navigation matured, Mr. Tazartes developed the architecture for the fiber-optic-gyro (FOG) based systems. He developed sophisticated closed-loop control of FOGs for vastly improved performance and contributed to the introduction of FOGs and micro-machined accelerometers into miniature Inertial Measurement Units (IMUs) and high accuracy navigation systems.

Currently, Mr. Tazartes is responsible for the development of next generation navigation technologies. He is a member of the Institute of Navigation and of the Institute of Electrical and Electronic Engineers (IEEE) and actively participates in the development of standards for inertial instruments and systems. Mr. Tazartes received his Master of Science in Electrical Engineering from the California Institute of Technology and his Bachelor's Degree in Science, Summa Cum Laude in Physics from the University of California at Los Angeles.

Mr. Tazartes is the recipient of the Institute of Navigation's 2002 Captain Phillip Van Horn (P.V.H.) Weems Award for "Continuing Contributions to the Art and Science of Navigation", the Engineers' Council 2000 Distinguished Engineering Achievement Award, and Litton Industries Advanced Technology Achievement Awards in 1992, 1995, and 2000.



### **Dr. John Mark**

Dr. John Mark is a recognized international authority on inertial and multi-sensor navigation and a pioneer in the field of strapdown inertial systems. Dr. Mark did his undergraduate work at the Massachusetts Institute of Technology and received his Doctor of Philosophy, Applied Mathematics, from the University of Southern California in 1973.

Over his career at Northrop Grumman Navigation Systems Division and at the former Litton Guidance and Control Systems Division, Dr. Mark has been at the forefront of navigation technology and has made outstanding contributions in all areas in the field. He holds over forty-five patents which cover all aspects of navigation systems. He has pioneering patents in strapdown methods, dither control of Ring Laser Gyros (RLGs), correction of RLG errors, multi-oscillator gyro designs, fiber optic gyro designs, low noise electronics, fiber optic gyro control, spread spectrum modulation, system compensation, altitude damping, and nuclear magnetic resonance devices. He has been published in over two dozen technical journals and textbooks. Dr. Mark developed the first Litton strapdown navigation algorithms using highly efficient real-time implementations and demonstrated one nautical mile per hour flight performance in 1975 using tuned rotor gyros.

Dr. Mark extended this work to RLGs and multi-oscillator gyros throughout the 1980's enabling operation of strapdown navigation systems in very highly dynamic environments. The code, algorithms, and techniques developed at that time continue to be used today. In the late 1980's, with the advent of Global Positioning System (GPS), Dr. Mark was heavily involved in the development of embedded GPS-inertial systems. Since the 1990's, Dr. Mark has been instrumental in developing the first production fiber optic gyro systems as well as high performance redundant inertial measurement units for control of inherently unstable airframes. Despite his retirement, Dr. Mark continues to serve as a consultant to Northrop Grumman on navigation systems development.

---

## Abstract

*Ring laser gyroscopes are rate-integrating devices which accurately track angle changes with infinite resolution. However, most readout mechanisms used in these instruments detect interference fringes to provide a digital output representative of a quantized angle change. Typically, the quantization level (determined by the path length and area of the instrument) is on the order of a second of arc.*

*In the past, two techniques have been used to improve the readout resolution. The first uses a direct readout of the gyro interference pattern intensity but requires precise analog electronics and is easily degraded by noise. The second utilizes a phase-locked-loop (PLL) to effectively multiply the gyro fringe frequency. However, this requires a complex PLL which must track rapidly varying frequencies over a very wide range of frequencies.*

*The new approach discussed here is based on digital signal processing using the basic quantized data output from the fringe detectors. This technique is particularly applicable to the multi-oscillator laser gyro (ZLG™) which is a true strapdown (non-dithered) sensor capable of extremely low noise in pointing and tracking applications. This approach requires only a simple digital circuit and has excellent noise immunity, bandwidth, and accuracy. Effective resolution is enhanced by a factor of more than one hundred with the data remaining in purely digital form. This enhancement is accomplished by sampling the fringe counter at a high rate (typically 10 MHz) and performing a moving average and accumulation.*

## Introduction

Most modern navigation systems employ ring laser gyros of either a two-mode (conventional Ring Laser Gyro (RLG)) or four-mode (multi-oscillator or ZLG™) variety. These gyros are rate-integrating and normally output incremental angles in digital form with a one arc-sec typical quantization. This resolution is generally adequate for navigation purposes but induces substantial noise if the gyro is used for other purposes such as pointing or angular rate measurement. The Zero-Lock gyro (ZLG™) is optically biased and does not require dither. If quantization effects are removed, the ZLG™ is extremely well suited for low noise pointing and tracking applications

as well as for precision rate output. Most previous attempts at removing quantization have used analog techniques to extract additional information from the optical interference pattern. While these have been demonstrated, in general they require precision wide-band analog electronics and are highly susceptible to noise contamination and the other usual difficulties. The technique discussed below is a purely digital method of improving the gyro resolution. The discussion focuses on the ZLG™ but the technique can easily be adapted to conventional ring laser gyros as well.

## ZLG Fundamentals

All optical gyroscopes are based on the Sagnac effect which deals with the propagation of light in a rotating reference system. Due to the rotation, the frame is not inertial and a general relativistic analysis applies. A simple view of the Sagnac effect consists of the following. Consider a closed path created by a cavity or waveguide in which counter-propagating (clockwise and counter-clockwise) beams are circulating. When the system is rotating, the effective optical path “seen” by the beam that is traveling in the same direction as the rotation becomes longer while the path “seen” by the beam traveling in the opposite direction becomes shorter. The difference in the path lengths is a measure of rotation.

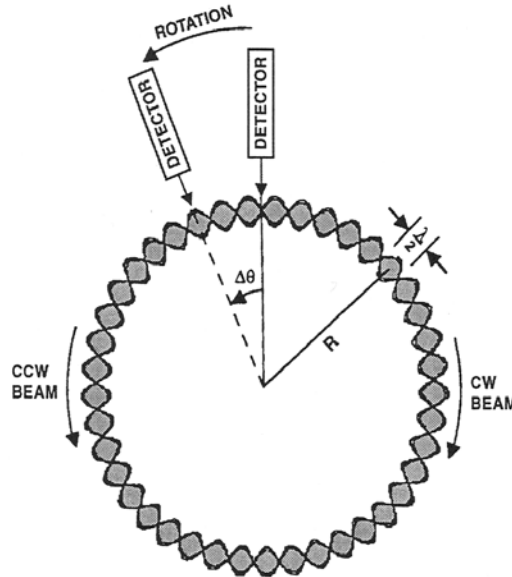
In a resonator (such as the various forms of RLGs), the clockwise and counter-clockwise beams form resonant modes in a cavity. These modes generate a standing wave pattern which exists in the cavity. Due to the Sagnac effect, the standing wave remains fixed in inertial space thereby providing a reference for measuring rotation. When the gyro is rotated, its body turns relative to the standing wave and a measure of the rotation is obtained by observing intensity changes as sensed by a detector fixed with respect to the gyro body. The operation of the ring laser gyroscope is best illustrated in Figure 1 which depicts a fictitious circular RLG. In this figure, a virtual standing wave pattern is created by the two counter-propagating waves. When the gyro is rotated, the detector moves with respect to the interference pattern and dark and light areas will alternate. Each dark/light cycle represents one half wavelength of the laser beam along the circumference of the path. The number of dark/light transitions can therefore be geometrically related to the angle of rotation as

indicated in Figure 1. By counting the number of fringes traversed, a measure of the total rotation angle is obtained. The scale factor of the instrument is based on purely geometrical relationships as given in Figure 1 for the “circular” RLG. A similar analysis can be made for any polygonal gyro. The scale factor remains geometrical, and depends on the ratio of enclosed area to path length according to the following expression:

$$N_{fringes} = \frac{4A}{\lambda L} \Delta\theta$$

- where: A = Area of gyro path projection on plane normal to input axis  
 L = Path length of gyro  
 $\lambda$  = Wavelength of laser  
 $\Delta\theta$  = Rotation angle increment

In a conventional RLG, the resonant cavity is designed to support only two counter-propagating linearly polarized modes essentially leading to a single standing wave. In a multi-oscillator gyro such as the ZLG™, two counter-propagating left hand circularly polarized (LCP) and two right hand circularly polarized (RCP) modes are supported. To avoid lock-in, an optical bias (known as the Faraday bias) is induced between the counter-propagating modes. The multi-oscillator gyro measures rate in much the same way as two independent gyros (one LCP and one RCP). When true rotation rate is applied to the multi-oscillator, the rate sensed by one of the two “gyros” increases while the rate sensed by the other “gyro” decreases. The two gyros can be characterized by their fringe accumulation equations:



- INTERFERENCE BETWEEN CW AND CCW BEAMS CREATES STANDING WAVE PATTERN
- STANDING WAVE PATTERN STAYS FIXED IN INERTIAL SPACE
- AS GYRO CASE ROTATES, DETECTOR MOVES AROUND RING AND COUNTS MINIMA
- SCALE FACTOR CORRESPONDENCE:

$\frac{\lambda}{2}$ CIRCUMFERENTIAL DISPLACEMENT	$\rightarrow \phi = 2\pi \text{ rad}$	OPTICAL PHASE SHIFT
$\Delta\theta$ MECHANICAL ROTATION	$\rightarrow \phi = \frac{4\pi R}{\lambda} \Delta\theta$	OPTICAL PHASE SHIFT

Figure 1. “Circular” RLG

$$F_L = 2\pi \left( f_F * t + \frac{4A}{\lambda L} \int_0^t \Omega(\tau) d\tau \right)$$

$$F_R = 2\pi \left( f_F * t - \frac{4A}{\lambda L} \int_0^t \Omega(\tau) d\tau \right)$$

where:  $F_L$  is the fringe phase of the LCP gyro  
 $F_R$  is the fringe phase of the RCP gyro  
 $f_F$  is the Faraday bias frequency  
 $t$  is time  
 $\Omega$  is the angular rate of the gyro

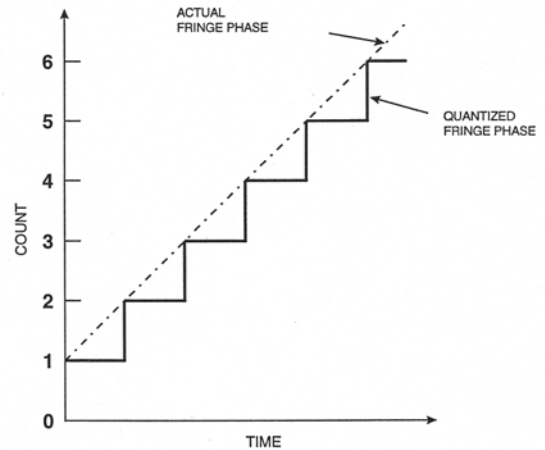
By subtracting the outputs of the two “gyros”, it is possible to cancel the Faraday bias while *doubling* the true angular rate measurement:

$$F_L - F_R = \frac{8A}{\lambda L} \int_0^t \Omega(\tau) d\tau = \frac{8A}{\lambda L} \Delta\theta$$

## Quantization

The RLG interference pattern is a continuous measure of angular displacement. However, the digital RLG readout schemes usually count light to dark or dark to light transitions leading to finite angle resolution. Because the interference pattern accurately tracks angle changes, no information is lost in the long term, only an angle quantization error is incurred. This, however can cause excessive noise if the gyro is used to measure angular rate. Inherently, quantization is a deterministic process<sup>1</sup> as discussed below.

Figure 2 shows a plot of continuous and quantized fringe phase in a ZLG™ as a function of time. The difference between the two curves is the quantization error as shown in Figure 3. For a constant angular rate, the quantization error is a periodic sawtooth function. Each of the two fringe phases is quantized independently leading to a difference phase (corresponding to rotation angle) which behaves as in Figure 4. The resulting patterns preserve correct angle



**Figure 2. Gyro Fringe Phase**

information in the long term but clearly induce noise in the instantaneous representation.

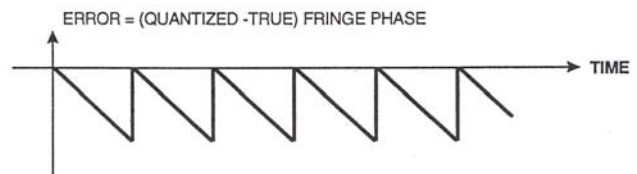
## Resolution Enhancement

Resolution enhancement is accomplished by reducing the quantization error through the use of high speed signal processing. Since the two fringe phases and quantization processes are independent and since the angle output is a linear function of the two (i.e. subtraction), each can be analyzed separately.

In the case of constant rate<sup>2</sup>, the quantization error shown in Figure 3 is easily decomposed into its spectral components:

$$\text{Error} = -\frac{1}{2} + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin 2\pi n ft$$

where:  $f$  is the fringe frequency of the gyro in question.

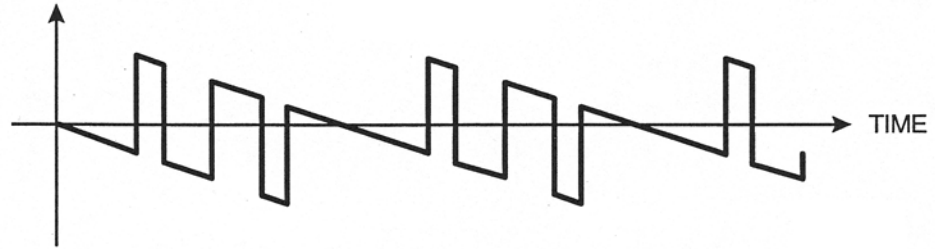


**Figure 3. Gyro Quantization Error**

<sup>1</sup> Under certain conditions, quantization is randomized and becomes equivalent to uniformly distributed white angle noise. Reference [1] discusses this situation in the context of gyro random walk measurement.

<sup>2</sup> The constant rate approximation is generally valid since the fringe frequency generated by input rate is by design much smaller than the Faraday frequency.

ZLG  
QUANTIZED FRINGE  
PHASE DIFFERENCE  
ERROR

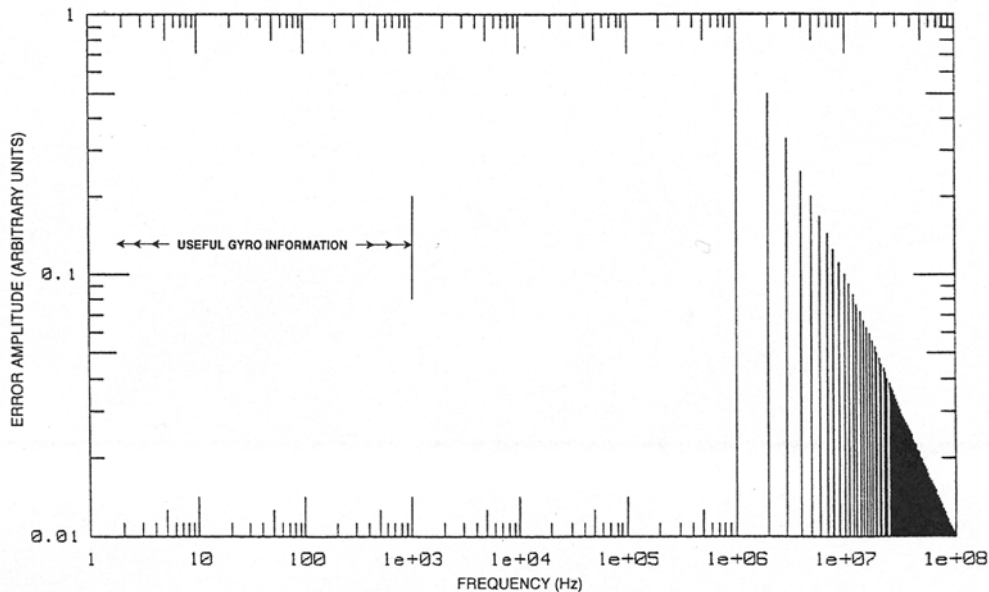


**Figure 4. ZLG Quantized Fringe Phase Difference Error**

Figure 5 is a pictorial diagram of the error spectrum. Because the Faraday frequency (typically one MHz) far exceeds the required angle tracking bandwidth of the gyro/system (typically one KHz), a low-pass filter can greatly attenuate the quantization error without degrading the useful gyro angle information. The digital output of the laser gyro lends itself well to digital filtering which can be supported by readily available circuits. However, it is recognized that the digital filter must have a high sampling rate in order to minimize aliasing.

It is well known that in sampled systems, frequencies above the Nyquist frequency (half the sampling frequency) are aliased into lower frequency bands. Once aliased, signals cannot be restored to their

original frequencies. Thus, any signals aliased into the filter pass-band clearly are not attenuated by the filter. Typically, for a Faraday frequency of one MHz, a sampling frequency of at least 10 MHz should be used. This ensures that no harmonics below the 10th will be aliased into the pass-band. The use of high sampling frequencies helps in two ways. First, only lower amplitude signals are aliased (the higher harmonics). Second, the probability of aliasing into the pass-band is reduced when operating on high harmonics. For example, if the sampling frequency is 10 MHz and the filter bandwidth is one kHz, then the 10th harmonic will alias into the pass-band only if the fringe frequency is within 100 Hz of one MHz.



**Figure 5. Quantization Error Spectrum**

While many types of digital filters may be used, it is essential that the filter be bit conservative (i.e. for every bit that enters the filter, an exact integer number of bits will exit) so that no net errors will be accumulated. Moving average filters have this exact property and have been used in the past (Reference [1]) for filtering quantization. These filters have the added advantage of simplicity of implementation as will be discussed below.

Figure 6 shows how laser gyro data is normally processed to form an incremental angle ( $\Delta\theta$ ). In Figure 7, a moving average filter has been added to reduce the quantization noise as discussed above. In a ZLG™, the two gyro outputs can be processed separately and combined after filtering. However, since the moving average is a linear filter, the quantized signals can be combined and then filtered thereby permitting the use of a single filter and sum-down as shown in Figure 8.

For ease of implementation, the length of the moving average is made identical to the length of the sum-down. In this case, the combination of the filter and sum can be reduced to a simple form which obviates the need for long circular buffers normally required for moving averages. Figure 9 is a block

diagram of the gyro data processing for this form. It can be shown that this implementation is exactly bit-for-bit equivalent to a moving average followed by a sum-down.

### Math Model

If the sampling frequency used in Figures 6, 7, 8 and 9 is sufficiently high (i.e. at least twice the highest fringe frequency), the gyro detection and quantization process along with the filter and sum-down can be represented as shown in Figure 10. It is assumed for convenience and simplicity that the moving average has the same length as the sum-down. The transfer functions are shown with a combination of Laplace and Z-transform operators. Figure 11 is a plot of the moving average filter amplitude response using 10 MHz sampling and a length of  $N = 10,000$  (equal to the sum-down length). The sum-down from 10 MHz to one kHz has the same bandwidth as the moving average filter and will therefore pass all frequencies within the filter pass-band including those resulting from aliasing. It is noted that it is only harmonics above 10th which can alias and these have amplitudes no more than 1/10th of the initial quantization error.

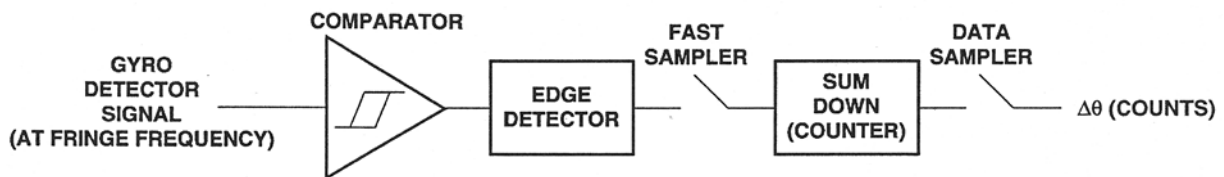


Figure 6. Laser Gyro Conventional Data Processing

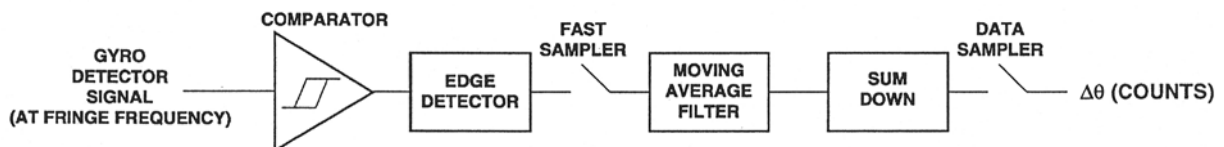


Figure 7. Laser Gyro Data Processing With Filter

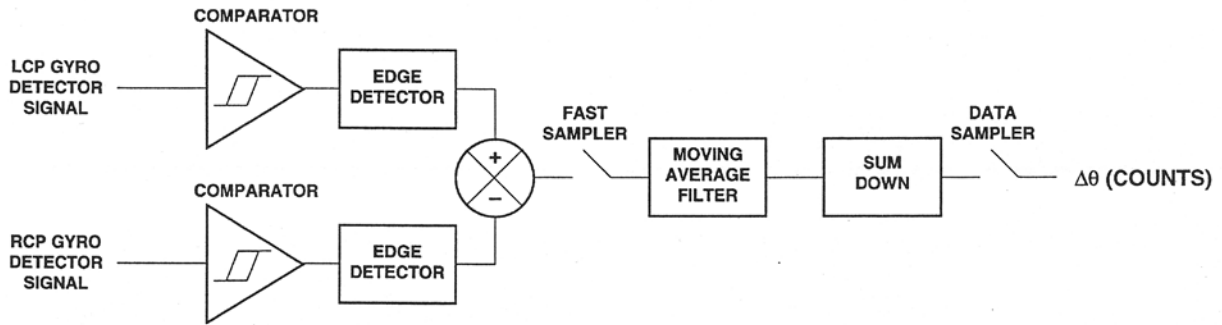


Figure 8. ZLG Data Processing With Filter

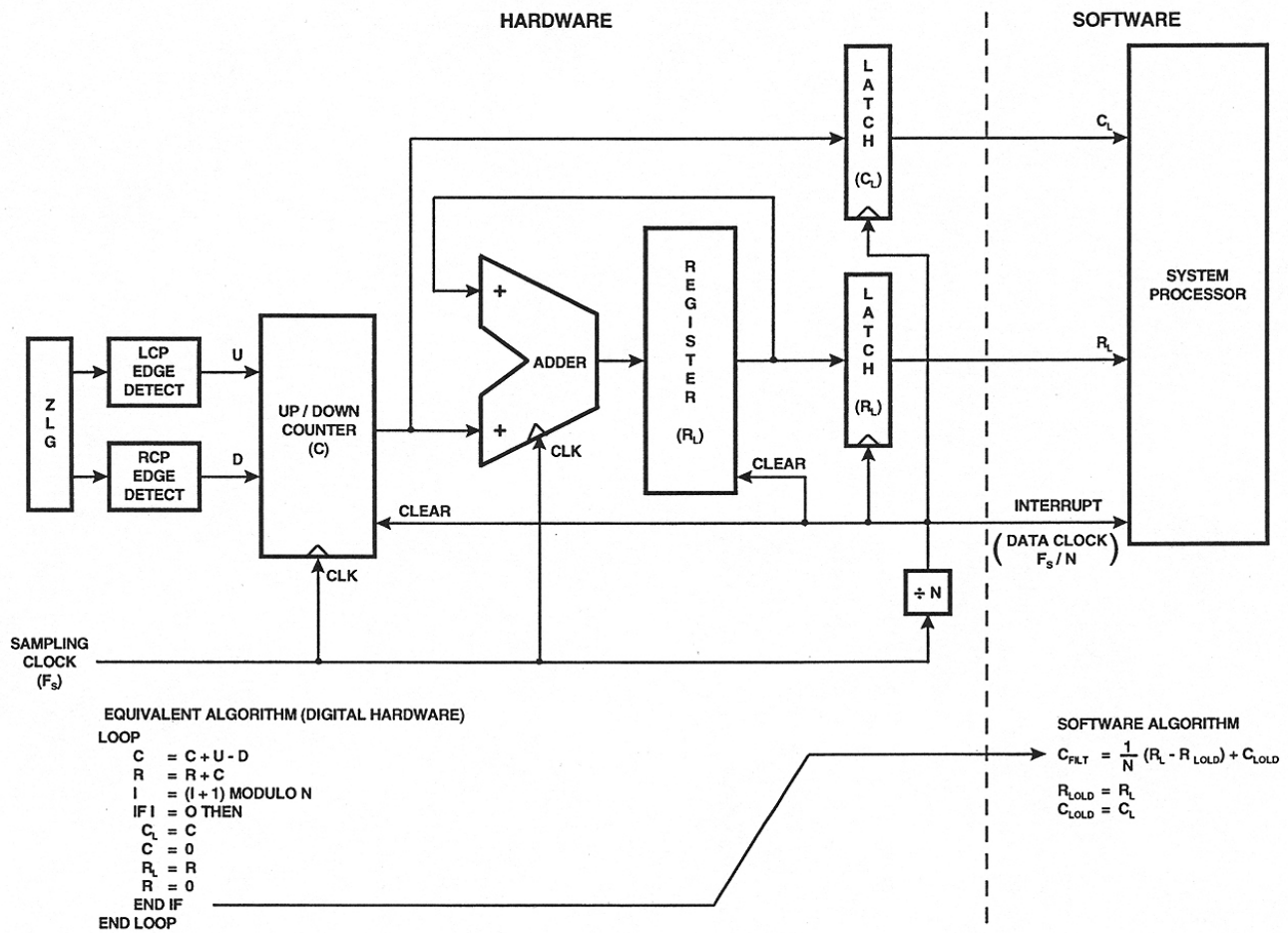
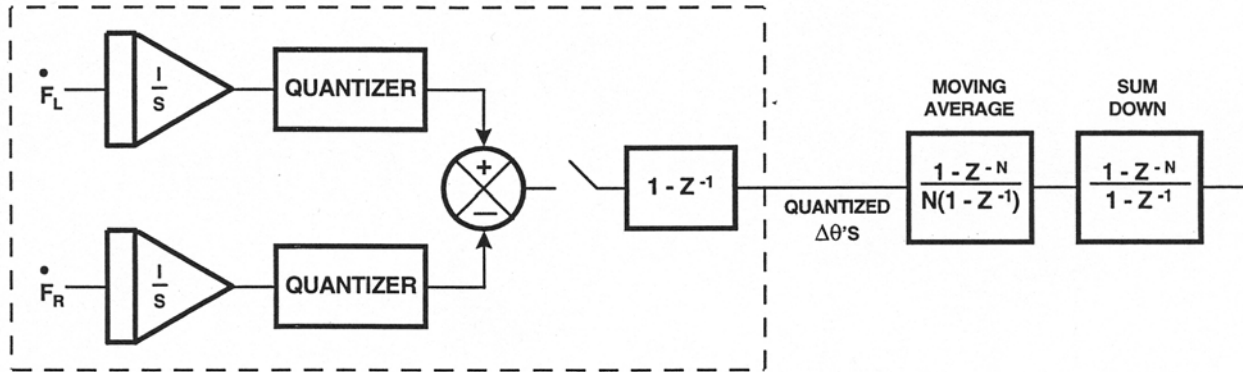
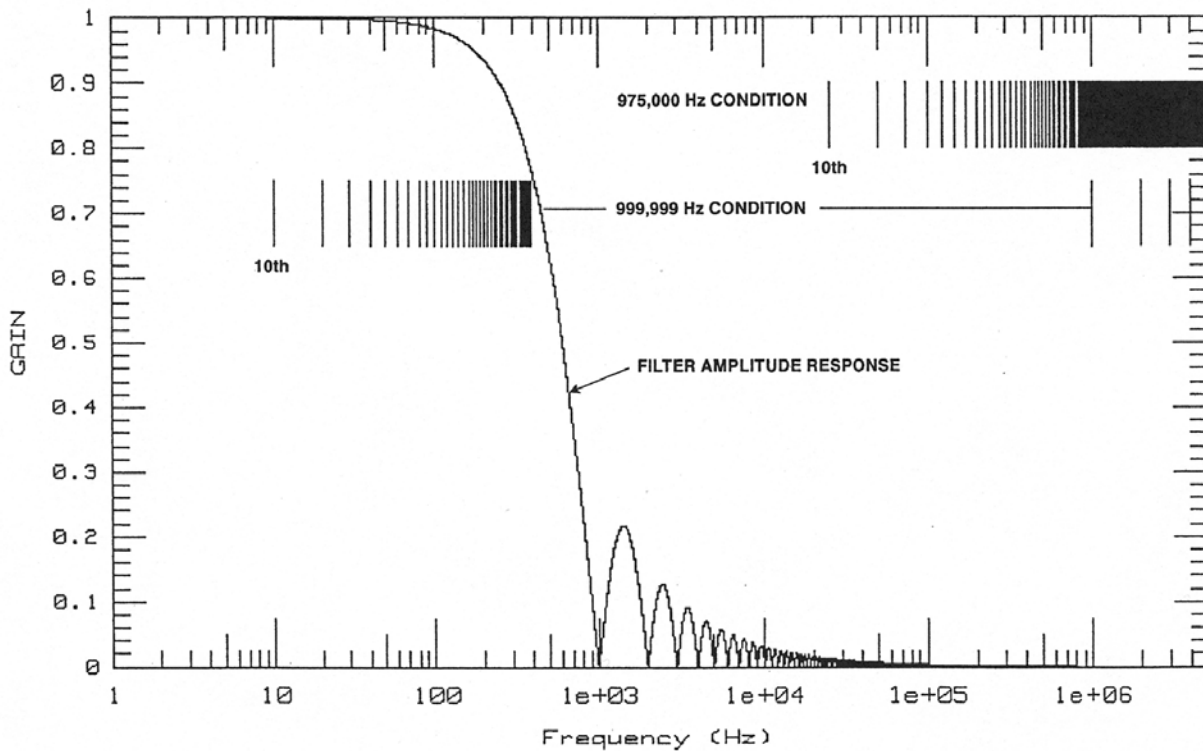


Figure 9. Resolution Enhancement Block Diagram



**Figure 10. Math Model for Resolution Enhancement**



**Figure 11. Resolution Enhancement Frequency Response**

Figure 11 also shows the location of the first 400 quantization error spectral lines assuming a fringe frequency of 975,000 Hz. In this case, it is clear that no harmonic at least below the 400th aliases into the filter pass-band. Thus, the quantization error will be reduced by a factor of at least 400. On the other hand, a second, more problematic case is also illustrated in Figure 11. In this latter instance, the fringe frequency is 999,999 Hz. While the first nine harmonics do not alias into the pass-band, the 10th harmonic and multiples of it survive virtually unattenuated through

aliasing. The aliased harmonics contain approximately 1/10th of the quantization error. Furthermore, due to the aliasing process, the quantization noise appears at low (undesirable) frequencies. The effectiveness of the filtering technique depends strongly on the exact fringe frequency as illustrated by the two extreme examples shown in Figure 11. Since the Faraday frequency which principally determines the fringe frequency is variable, any aliasing condition is possible resulting in variable residual quantization characteristics.

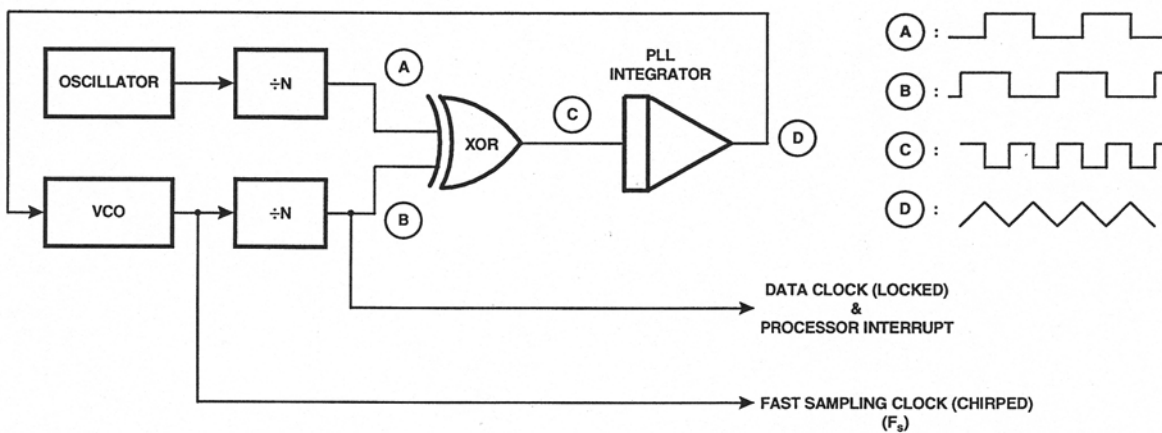
## Chirp

Chirp is an enhanced method for achieving uniform quantization error behavior. As shown above, the simple moving average filter is very effective at reducing quantization noise on average but its performance can be degraded by aliases of harmonics in the sampling process. Phase noise in the gyro detectors and noise in the comparators are insufficient to substantially break-up the patterns resulting from aliasing. One method of avoiding the performance degradation attributed to aliasing consists of continuously sweeping the Faraday frequency in order to smear the deterministic frequencies. However, this is usually impractical as it requires instrument modifications. An alternate method involves sweeping the sampling clock by a few percent (sampling chirp) with the same effect. The sampling chirp spreads the energy of each aliased harmonic into a wide frequency band so that only a very small fraction remains within the filter pass-band. This technique, combined with the comparator noise, effectively reduces the quantization error to a random process similar to that described in Reference [1]. In order to ensure proper long-term operation, the chirped sampling clock must have an exact and stable average frequency. This can be accomplished by phase-locking to a fixed lower frequency clock as shown in Figure 12. In this case, the properties of the phase-locked-loop provide chirp in the form of a sampling frequency which varies as a triangle wave. The number of sampling clock strobes generated within one data output interval is guaranteed to be constant (exactly  $N$ ) because of the phase lock

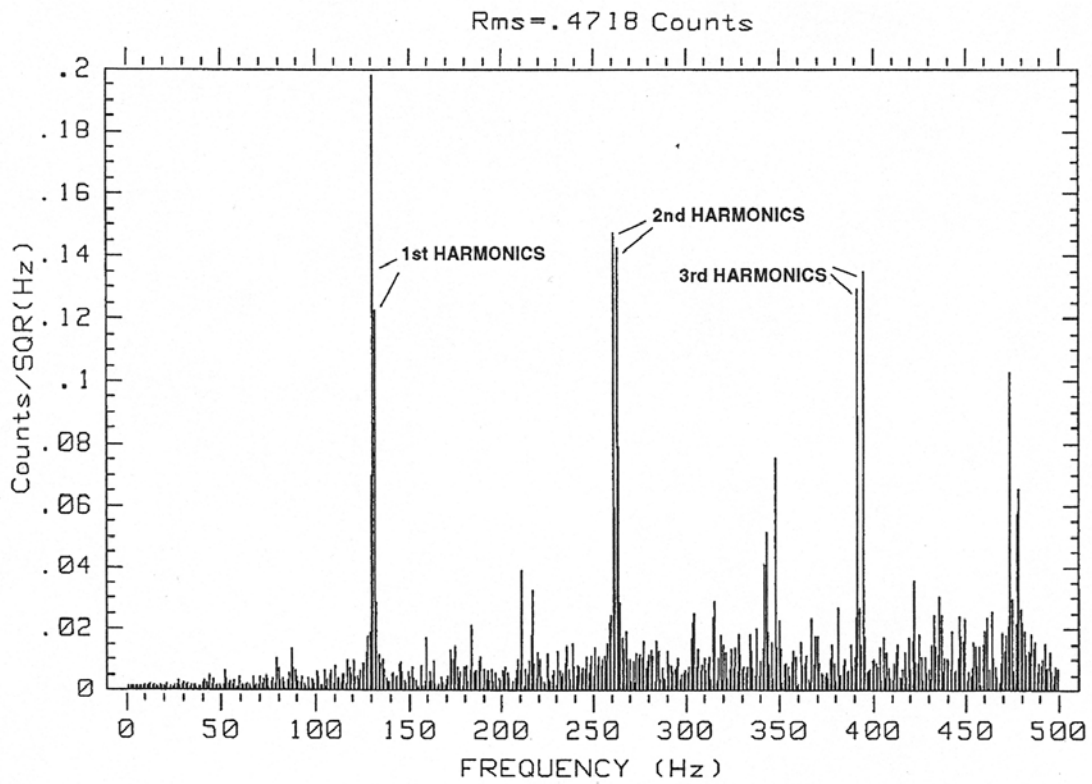
implemented at the data output frequency. Reference [2] provides additional information on resolution enhancement with a chirped clock.

## Results

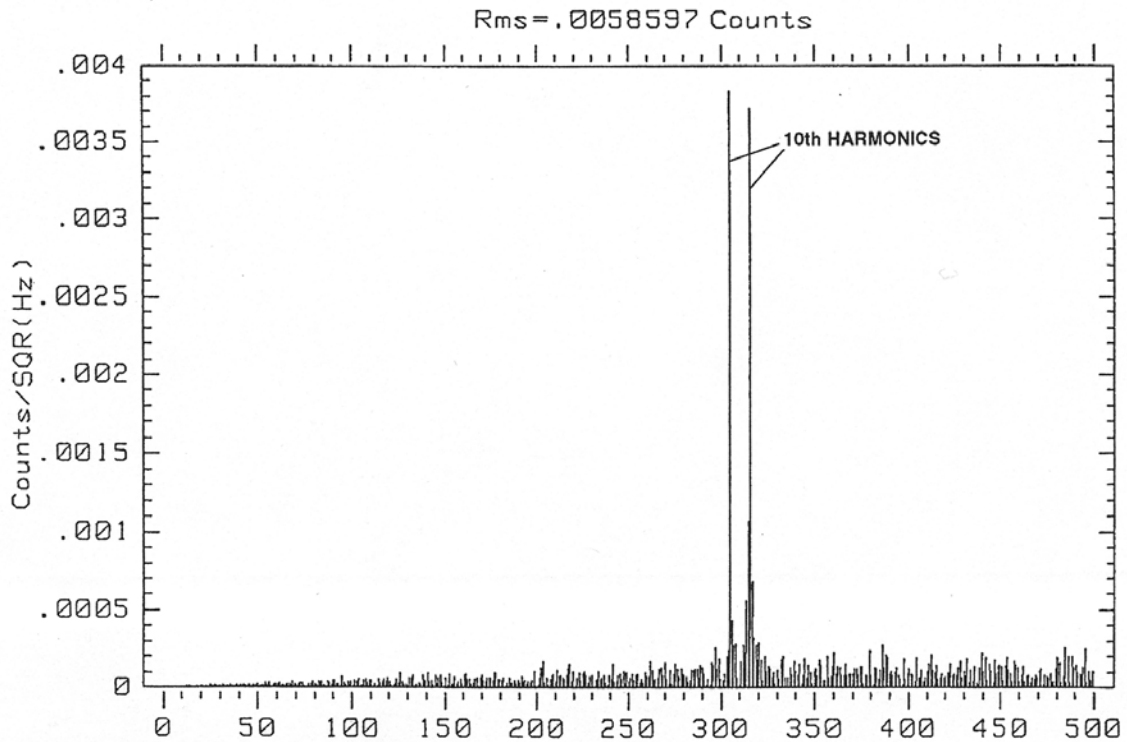
A simulated ZLG™ output was created and applied to the digital resolution enhancement technique. A LCP fringe frequency of 1,000,130.476 Hz and an RCP frequency of 1,000,131.524 Hz were selected. The sampling frequency is 10 MHz and the data frequency is one kHz. Figure 13 shows the spectrum of the *unfiltered* quantization noise data. The aliases of the quantization harmonics are clearly visible in this plot, starting from the first harmonics aliasing to 130.476 and 131.524 Hz, the second harmonics at 260.952 and 263.047 Hz, and continuing upward. Figure 14 shows the spectrum of the *filtered* data. The root-mean-square noise has been dramatically reduced as compared to the unfiltered data but the alias of the 10th quantization harmonics are clearly visible in the spectrum. These appear at the aliases of 10,001,304.76 Hz and 10,001,315.24 Hz into the one kHz band, hence at 304.76 and 315.24 Hz. In order to “smear out” these remaining harmonics, a chirp of +/- 400 kHz is applied to the sampling frequency. The resulting error spectrum, shown in Figure 15, no longer exhibits discrete frequencies and resembles the characteristics of angle noise at a much lower quantization level. The root-mean-square error has been reduced by a factor of approximately 250 as compared to the unfiltered case. Results of resolution enhancement tests on actual instruments may be found in Reference [3].



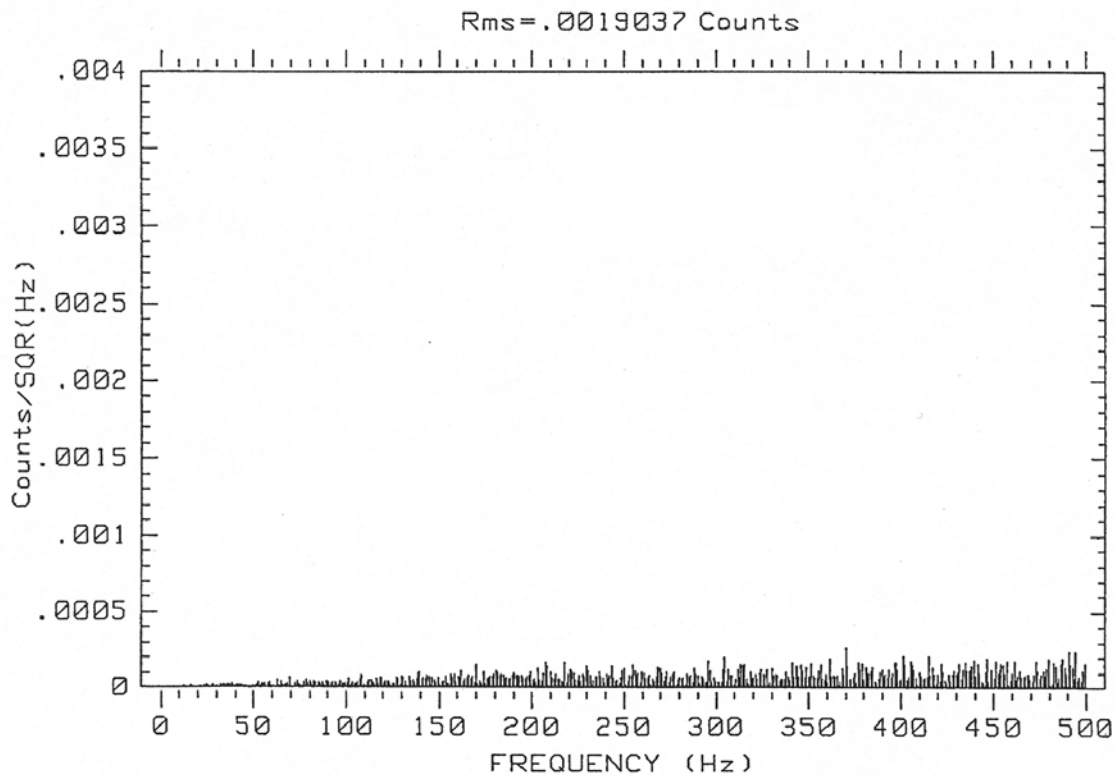
**Figure 12. Sampling Clock Chirp Generator**



**Figure 13. Noise Content of Unfiltered Data Example**



**Figure 14. Noise Content of Filtered Data Example**



**Figure 15. Noise Content of Chirped Filtered Data Example**

## Conclusion

A resolution enhancement method for laser gyros has been described. This very effective technique can be implemented with simple digital circuits. For use with ultra-low noise gyroscopes such as the ZLG™, an additional feature has been incorporated to eliminate aliasing in the high speed sampling process. A simple phase-locked-loop is used to vary the instantaneous sampling frequency while maintaining an exact average frequency. The technique discussed above has been demonstrated to reduce root-mean-square quantization noise by a factor greater than 100 and has been used in several Northrop Grumman products. The resolution-enhanced ZLG™ is ideally suited for low noise angular rate measurements or for pointing applications.

## References

- [1] "Laser Gyroscope Random Walk Determination Using a Fast-Filtering Technique", J. Mark, A. Brown, Proceedings: Symposium Gyro Technology, Stuttgart, Germany, 1984
- [2] "Ring Laser Gyroscope Enhanced Resolution System", U.S. Patent 5,485,273, Mark et al, January 16, 1996
- [3] "Zero-Lock Laser Gyro", M. Fernandez, R. Ebner, N. Dahle, Proceedings: 12th The American Aeronautical Society's Annual Guidance and Control Conference, February 4-8, 1989, Keystone Colorado

---

## **Strategic Programs & Business Development (SP&BD), Navigation Systems Division**

One of the main functions of the SP&BD organization is to sustain NSD's competitive advantage in all of its product lines by providing cutting edge technologies that supply entirely new capabilities to the marketplace and also enhance its current products.

**Ike J. Song, Director of SP&BD**  
**ike.song@ngc.com**

*For more information, please contact:*  
Northrop Grumman Corporation  
Navigations Systems  
21240 Burbank Boulevard  
Woodland Hills, CA 91367 USA  
1-866-NGNAVSYS (646-2879)  
[www.nsd.es.northropgrumman.com](http://www.nsd.es.northropgrumman.com)



***NORTHROP GRUMMAN***